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DISTRIBUTED MATRIX COMPLETION APPLICATION TO COOPERATIVE POSITIONING IN NOISY ENVIRONMENTS

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14. ABSTRACT
The PI and collaborators developed novel algorithms for positioning, building on earlier developments in matrix completion and high-dimensional statistics. In particular, a distributed version of matrix completion-based positioning, and a gossip version of low-rank approximation were developed. A convex relaxation for positioning in the presence of noise was shown to be constant-optimal.
Additional contributions were made in several other areas: Finding dense substructures of large networks in nearly linear time; Approximate message passing algorithms and in particular their application to spatially-coupled compressed sensing; MEasures of statistical significance in high dimension.

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Distributed Matrix Completion:
Applications to Cooperative Positioning in Noisy Environments

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December 5, 2013

Outline

Below is an outline summary of the main scientific results that have been funded within this project:

1. *Distributed algorithms for positioning and low-rank approximation.*
   Publications [KMO11, MO10].
2. *Positioning via convex optimization.*
   Publications [JM11, JM13c].
3. *Approximate message passing algorithms.*
   Publications [DMM11, BM11, BLM12, DJM13, DGM13, JM12].
4. *Finding highly connected/atypical subnetworks.*
   Publications [DM13].
5. *Assessing uncertainty in high dimensional statistics.*
   Publications [JM13b, JM13a].

The main collaborators in this research have been Mohsen Bayati (Stanford University), Yash Deshpande (graduate student, Stanford University), David Donoho (Stanford University), Morteza Ibrahimi (graduate student, Stanford University), Adel Javanmard (graduate student, Stanford University). Satish Korada (postdoc, Stanford University). The work of Deshpande, Ibrahimi, Javanmard, Korada was partially supported through this grant.

All publications are available through on leading journals/conference proceedings. Publications under review are made available online through arxiv and through the PI's webpage. The next sections provide pointers to the main results.

Distributed algorithms for positioning and low-rank approximation

The basic question in matrix completion is to infer a large low-rank matrix from a small subset of its entries. Positioning refers to the task of inferring the locations of n points from a subset of their

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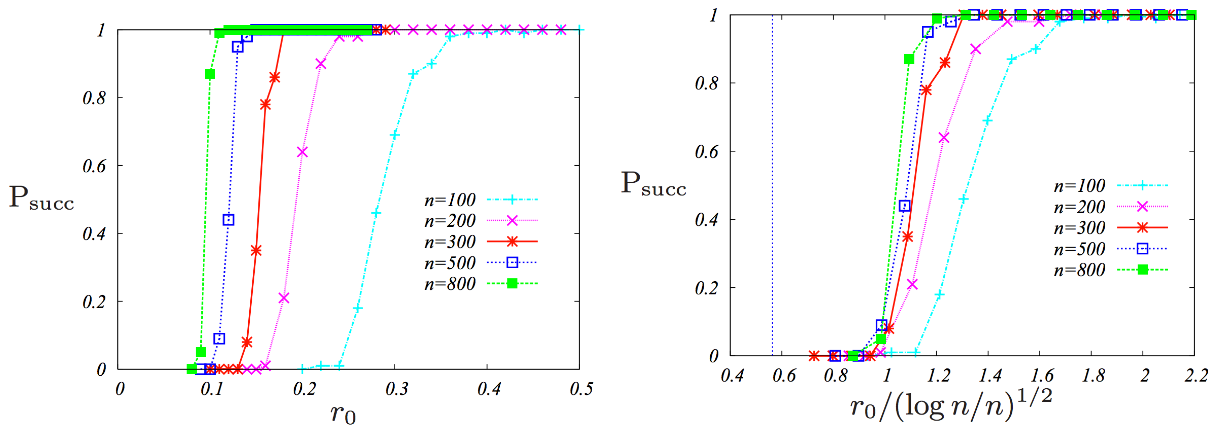

Figure 1: Success probability of OPTSPACE POSITIONING as a function of the measurement range r_0, for various network sizes: n nodes are placed uniformly in the unit square $[1,1]^2$. On the right, r_0 is divided by the connectivity scale $r(n) = \sqrt{(\log n)/n}$. The vertical line marks the onset of connectivity.

distance. It turns out that positioning can be viewed as a matrix completion problem, although of a peculiar type. The paper [MO10] develops an algorithm for positioning using ideas from matrix completion, cf. Fig. 1. A distributed implementation is also demonstrated.

Many algorithms that compute positions of the nodes of a wireless network on the basis of pairwise distance measurements require a few leading eigenvectors of the distances matrix. One example is MDS-MAP. While eigenvector calculation is a standard topic in numerical linear algebra, it becomes challenging under severe communication or computation constraints, or in absence of central scheduling.The paper [KMO11] investigates the possibility of computing the leading eigenvectors of a large data matrix through gossip algorithms. A new algorithm is proposed that amounts to iteratively multiplying a vector by independent random sparsification of the original matrix and averaging the resulting normalized vectors. This can be viewed as a generalization of gossip algorithms for consensus. The algorithms outperform state-of-the-art methods in a communication-limited scenario.

Positioning via convex optimization

In presence of noise, maximum likelihood localization is a hard non-convex optimization problem. The papers [JM11, JM13c]. propose a reconstruction algorithm based on semidefinite programming. This is the first algorithm of this type for which tight performance guarantees have been proved. For a random geometric graph model and uniformly bounded noise, these papers establish a precise characterization of the algorithm's performance. In particular, in the noiseless case, there exists a connectivity radius r_0 beyond which the algorithm reconstructs the exact positions (up to rigid transformations). In the presence of noise, the papers establish upper and lower bounds on the reconstruction error that match up to a factor that depends only on the dimension d, and the average degree of the nodes in the graph.

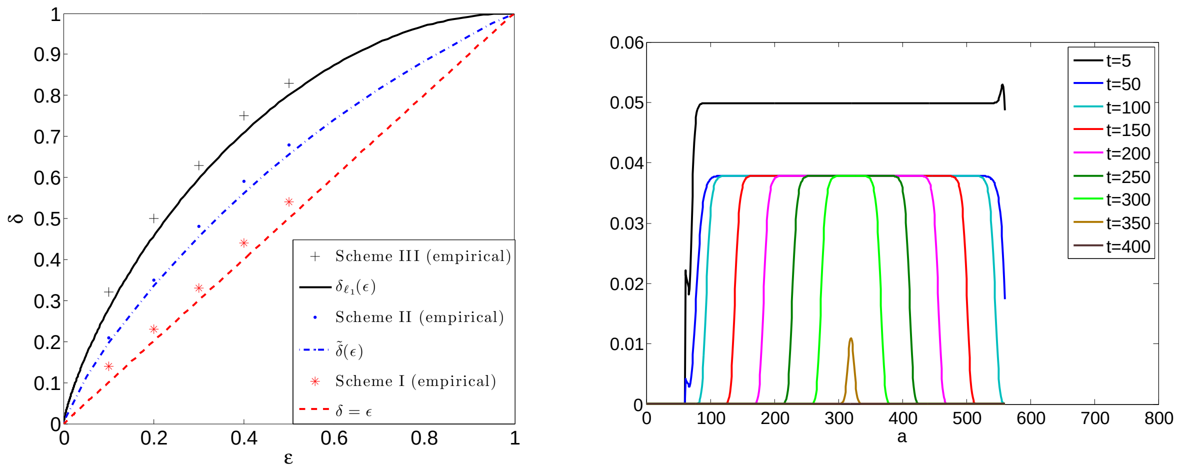

Figure 2: Left: phase transitions for several compressed sensing schemes. Scheme I: Standard compressed sensing with dense partial Fourier matrices and convex optimization-based reconstruction. Scheme II: dense partial Fourier matrices and Bayes-optimal AMP reconstruction. Scheme III: 'spatially coupled' partial Gabor matrices and Bayes-optimal AMP reconstruction. Right: Evolution of the mean square reconstruction error across the signal, as AMP iteration proceeds.

Approximate message passing algorithms

Approximate message passing (AMP) algorithms were developed in [DMM09] as a way to solve certain compressed sensing reconstruction problems. The basic idea is to define a graphical model that is associated with the problem of interest, and to apply methods for approximate inference in graphical models, and in particular message passing algorithms inspired by belief propagation. Often the resulting graph is dense which is at odds to the standard wisdom suggesting that this class of algorithms is most effective on sparse graphs.

It was soon realized that the same approach can be applied to a host of other statistical estimation problems (see [Mon12] for a brief overview and next section for a specific example). Further, the theory developed in [DMM09, DMM11, BM11, BLM12] provides a sharp asymptotic analysis of such algorithms. This analysis shows that AMP is extremely effective on some classes of dense graphs and, furthermore, dense graphs bring along special simplifications that can reduce the computational complexity with respect to sparse cases.

Compressed sensing with 'spatially coupled' sensing matrices provides a spectacular application of this approach. The papers [DJM13, JM12] show that such a scheme can effectively solve the reconstruction problem with undersampling rates close to the fraction of non-zero coordinates. For sparse signals, i.e., sequences of dimension n and $k(n)$ non-zero entries, this implies reconstruction from $k(n) + o(n)$ measurements. For 'discrete' signals, i.e., signals whose coordinates take a fixed finite set of values, this implies reconstruction from $o(n)$ measurements. The result is robust with respect to noise and does apply to non-random signal.

This phase transition phenomenon survives for 'spatially coupled' matrices with considerable amount of structure. In particular, the paper [JM12] studies the problem of reconstructing signals that ase sparse in Fourier domain, from subsampled Gabor transform. The results are illustrated in Fig. 2.

Finding highly connected/atypical subnetworks

Numerous modern data sets have network structure, i.e. the dataset consists of observations on pairwise relationships among a set of n objects. A recurring computational problem in this context is the one of identifying a small subset of 'atypical observations against a noisy background. The motivation can be –for instance– to find a tightly connected small community in a large social network.

The paper [DM13] develops a new type of algorithm and analysis for this problem. In particular, the new algorithm improves over the best methods for nding a hidden clique in an otherwise random graph, a special problem that attracted substantial interest within theoretical computer science.

The new algorithm is based on a different philosophy with respect to previous approaches to the same problem. It aims at estimating the hidden set by computing, for each vertex in the network, the posterior probability that it belongs to the hidden set, given the observed data.

This is, in general, an intractable problem. We therefore consider a message passing algorithm derived from belief propagation, a heuristic machine learning method for approximating posterior probabilities in graphical models. We develop a rigorous analysis of this algorithm that is asymptotically exact as $N \to \infty$, and prove that indeed the algorithm converges to the correct set of vertices if

$$\lambda|S| \geq \sqrt{\frac{N}{e}}\,(1+\epsilon)\,. \tag{1}$$

Here S is the hidden set, with size $|S|$, λ quantifies the difference between connections within and without the subnetwork, and finally ϵ is an arbitrary positive number.

Assessing uncertainty in high dimensional statistics

Fitting high-dimensional statistical models often requires the use of non-linear parameter estimation procedures. As a consequence, it is generally impossible to obtain an exact characterization of the probability distribution of the parameter estimates. This in turn implies that it is extremely challenging to quantify the uncertainty associated with a certain parameter estimate. Concretely, no commonly accepted procedure exists for computing classical measures of uncertainty and statistical significance as confidence intervals or p-values.

The papers [JM13b, JM13a] consider a broad class regression problems, and propose an efficient algorithm for constructing confidence intervals and p-values. The resulting confidence intervals have nearly optimal size. When testing for the null hypothesis that a certain parameter is vanishing, the new method has nearly optimal power.

The new approach is based on constructing a 'de-biased' version of regularized M-estimators. The new construction improves over recent work in the field in that it does not assume a special structure on the design matrix.

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